

FLUID MECHANICS

Fluid is the name given to a substance, which begins to flow when external force is applied on it.

Fluids exert thrust

This normal change in momentum transferred to the walls per unit time by the molecules of fluid accounts for the normal force or thrust of fluid on the walls of the container.

Liquid

The normal force (or thrust) exerted by a liquid at rest per unit area of the surface in contact with it is called pressure of liquid or hydrostatic pressure.

Density and relative density

Density of a substance is defined as the mass per unit volume of the substance, i.e.

$$\text{density, } \rho = \frac{\text{mass (m)}}{\text{volume (V)}} = \frac{m}{V}$$

Relative density of substance is defined as the ratio of its density to the density of water at 4°C,

$$\text{Relative density} = \frac{\text{density of substance}}{\text{density of water at 4°C}}$$

Variation of pressure with depth

The pressure is same at all points inside the liquid lying at the same depth in a horizontal plane.

$P - P_a = h\rho g$. It is called gauge pressure

Gauge pressure at a point in a liquid is the difference of total pressure at the point and atmospheric pressure.

Pressure exerted by a liquid column of height h is independent of area of cross-section A but depends upon the height h of the liquid column and density ρ of the liquid.

Pascal's law

It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same.

Principle of transmission of pressure in liquids or gases.

Pascal's law states that the increase in pressure at one point of the enclosed liquids in equilibrium of rest is

transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

Atmospheric pressure

The gaseous envelope surrounding the earth is called earth's atmosphere.

The atmospheric pressure at any point is equal to the weight of a vertical column of air of unit cross-sectional area extending from that point to the top of the earth's atmosphere.

Buoyancy

This upward force acting on the body immersed in a fluid is called upward thrust or buoyant force or simply buoyancy.

Archimede's Principle*

It states that when a body is immersed wholly or partly in a liquid at rest, it loses some of its weight. The loss in weight of the body in the liquids is equal to the weight of the liquid displaced by the immersed part of the body.

Law of Floatation

When a body of density ρ , volume V is immersed completely in a liquid of density σ , two forces are acting on it.

Case I. If $W > w$, then $W - w$ is positive. In this case, the body will sink to the bottom of the liquid. If the body is not hollow from inside, then the density of solid body is greater than the density of liquid (i.e. $\rho > \sigma$).

Case II. If $W < w$, then $W - w$ is negative. In this case, the body will rise above the surface of liquid to such an extent that the weight of the liquid displaced by the immersed part of the body.

Case III. If $W = w$, then $W - w = 0$. It means the resultant force acting on the body fully immersed in the liquid is zero. In this case, the body is at rest anywhere within the liquid. The apparent weight of the body is zero at all positions inside the liquid. In this situation, the body will float if its whole volume is just immersed in the liquid. If it is solid then the density of body is equal to the density of liquid (i.e., $\rho = \sigma$)

Thus the law of floatation states that a body will float in a liquid, if weight of the liquid



displaced by immersed part of the body is atleast equal or greater than the weight of the body.

Equilibrium of floating bodies if the following conditions are fulfilled.

- (i) A body can float if the weight of the liquid displaced by the immersed part of body must be equal to the weight of the body.
- (ii) A body can be in equilibrium if the center of gravity of the body and center of buoyancy must be along the same vertical line.

1. Definition of viscosity:

The property of a fluid by virtue of which it opposes the relative motion between its different layers is called viscosity.

2. Coefficient of dynamic viscosity:

The coefficient of viscosity of a liquid is the viscous force acting per unit area between two adjacent layers moving with unit velocity gradient.

3. Formula:

Newton's law of viscosity is given by

$$F = -\eta A \frac{dv}{dx} \text{ [negative sign indicates the oppositeness of } F \text{ and } V]$$

where

F = Viscous force

η = Coefficient of viscosity

A = Area of a layer

$$\frac{dv}{dx} = \text{Velocity gradient}$$

4. Formula

Poiseuille's equation is given by

$$V = \frac{\pi Pr^4}{8\eta L}$$

where

V = Volume of liquid flowing per sec

P = Pressure difference between the ends of the capillary tube.

L = Length of the capillary tube

r = Radius of capillary tube.
the axis of the tube

2. Velocity of flow must be less than critical velocity

3. Pressure across any cross-section is constant

4. Velocity of a liquid layer in contact with the walls of the tube is zero

Stoke's law

When a small spherical body moves through a viscous medium at rest, layers of the medium touching the body are dragged along with it. But the layers of the medium away from the body are at rest. This causes a relative motion between different layers of medium. As a result of this, a backward dragging force (i.e., viscous drag) comes into play, which opposes the motion of the body. This backward dragging force increases with the increase in velocity of the moving body.

5. Stoke's formula

a) Stoke's formula is given by

$$F = 6\pi\eta r V_t$$

Where F = Viscous force, V_t = terminal velocity

$$b) V_t = \frac{2}{9} r^2 \frac{(d_1 - d_2)g}{\eta}$$

Where

d_1 = density of solid

d_2 = density of liquid

6. Critical velocity

Critical velocity is defined as the minimum velocity at which the flow of a liquid changes from laminar (or streamline) to turbulent state is called critical velocity

$$V_c = \frac{R\eta}{\rho D}$$

Where

R = Reynold's Number

η = Coefficient of viscosity

ρ = Density

D = Diameter of the tube

7. Equation of continuity

Equation of continuity is given by

$$av = \text{const} \Rightarrow a_1 v_1 = a_2 v_2$$

Where

a = area and v = velocity

8. Energy of a Liquid



A liquid in motion may possess three types of energy

(i) **Pressure** It is the energy possessed by all liquid by virtue of its pressure. It is the measure of work done in pushing the liquid in the vessel against pressure without imparting any velocity to it.

(ii) **Potential Energy.** It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any reference level taken as zero level.

Potential energy per unit volume of liquid

$$= \frac{mgh}{V} = \rho gh$$
 where ρ is the density of liquid.

(iii) **Kinetic energy** It is the energy possessed by a liquid by virtue of its motion or velocity.

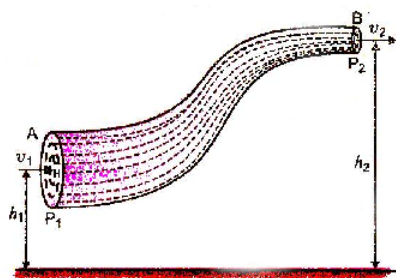
K.E. per unit volume of the

$$\text{liquid} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\rho v^2$$

Bernoulli's theorem

Bernoulli's theorem is an outcome of the principle of conservation of energy applied to a liquid in motion.

This theorem states that for the streamline flow of an ideal liquid, the total energy (the sum of the pressure energy, potential energy and kinetic energy) per unit mass remains constant every cross-section throughout the flow.



$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{a constant}$$

the sum of pressure energy, potential energy and kinetic energy per unit mass is constant at all cross-section in the streamline flow of an ideal liquid. This proves Bernoulli's theorem

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{a constant}$$

$$\text{for horizontal tube } P + \frac{1}{2}\rho v^2 = \text{const.}$$

Application of Bernoulli's Theorem

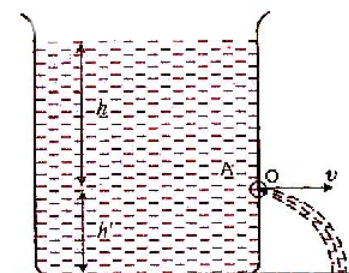
i. Atomiser or sprayer. ii. Lift on an aeroplane wing

iii. Blowing off the roofs during storm.

Venturimeter

It is a device, used for measuring the speed of incompressible liquid and rate of flow of liquid through pipes. Its working is based on Bernoulli's Theorem.

9. Torricelli's theorem



Torricelli's theorem is given by

$$a. V = \sqrt{2gh} \text{ where } V = \text{Velocity of efflux}$$

$$b. R = 2\sqrt{h(H-h)}$$

where

h = depth of orifice from free surface of liquid

H = total height of liquid

R = range of the liquid

10. Dynamic lift

Dynamic lift is given by

$$F = \frac{1}{2}d(V_1^2 - V_2^2)A$$

where

F = dynamic lift

V_1 = velocity above wing

V_2 = velocity below wing,

A = area of wing

d = density of air